Virtual SEA for Importing FEM Information in SEA Models

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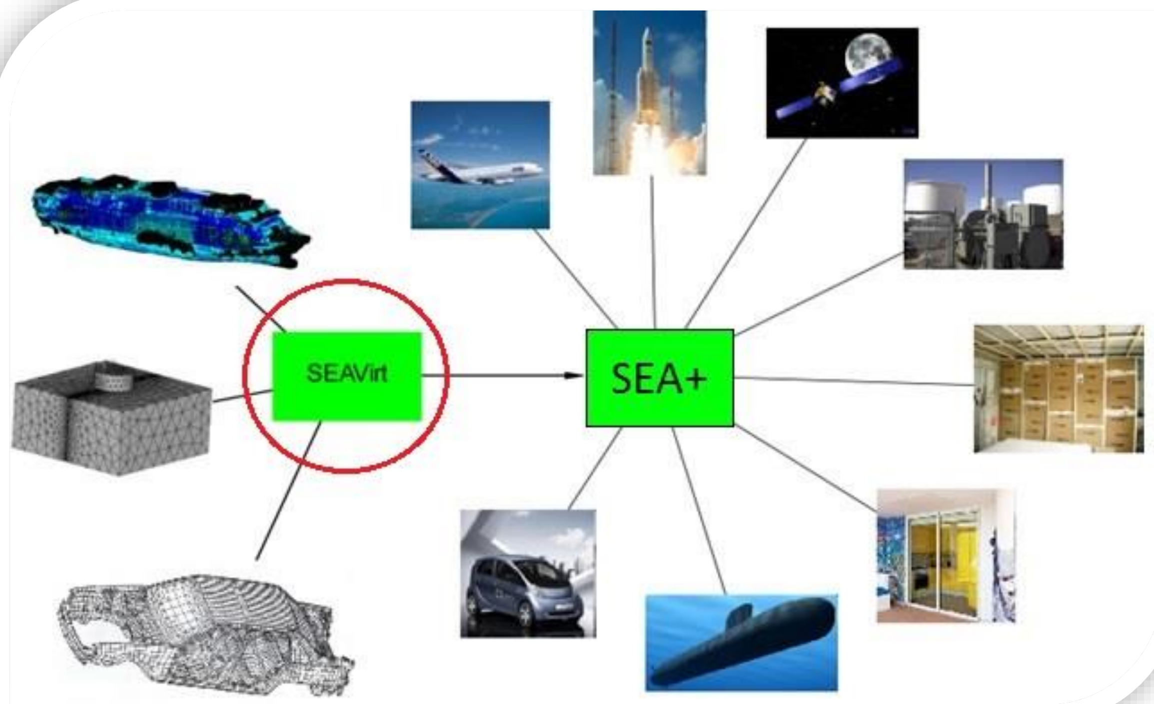
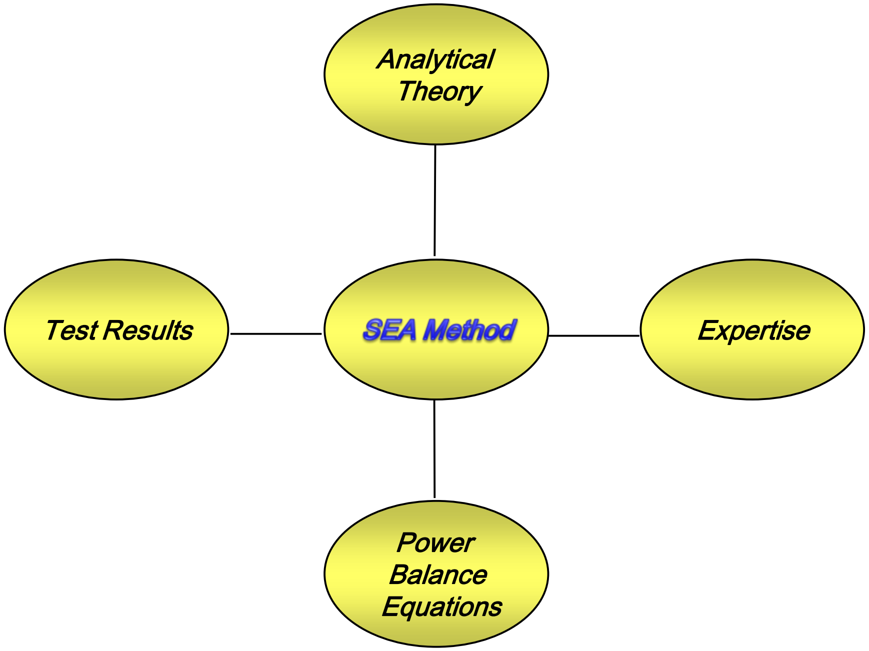
SEA Modeling Data Flow and Applications

- For developing **SEA** (**Statistical Energy Analysis**) models, **SEA+** software offers powerful scientific functions for high-end industrial applications (Aerospace, Automotive, Building, Defense, Energy, Mechanical, Railway…)
- SEA+ may predict SEA parameters from analytical formulations and from **FEM** (Finite Element Method) model through **Virtual SEA** (**VSEA**)
- **VSEA** technology is implemented in **SEAVirt** Module



Controlling SEA Confidence Levels

- To control SEA model accuracy, Analytical SEA (**ASEA**) was needed to be complemented by Experimental SEA test method (**ESEA**) also named Power Injection Method (**PIM**) to take into account junction energy (**indirect coupling**)
- ESEA provides all SEA parameters, Damping Loss Factors (**DLF**), direct and indirect Coupling Loss Factors (**CLF**) from Frequency Response Function (**FRF**) measurement on the built-up system
- InterAC has introduced ESEA to industry in the early 90th's, **SEA-XP** (former name AutoSEA-X) and now **SEA-TEST** software
- The various tasks required for SEA modeling are sketched hereafter



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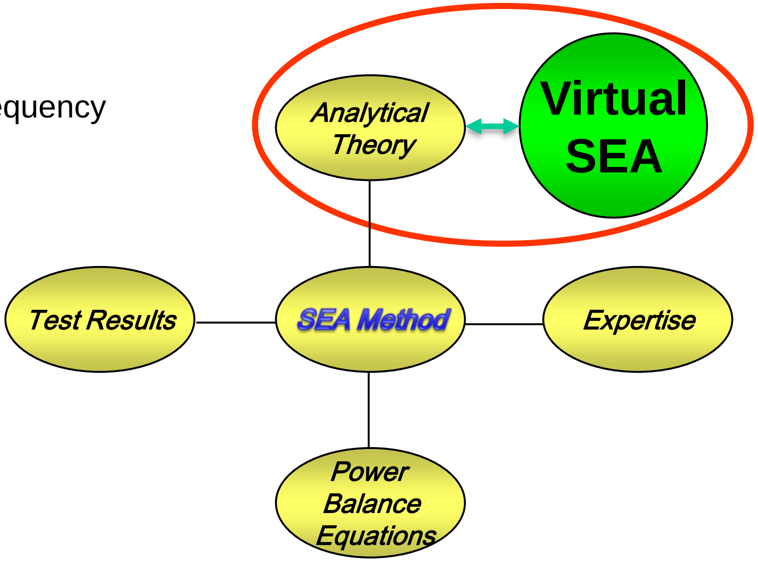
Hybrid Modeling with VSEA

- SEA parameters were proven to be reliably identified by ESEA. For structural models such as car body-in-white, ESEA models were found far more accurate than their ASEA representation with current modeling technique
  - ***Application of SEA to the analysis of vibration response of car cabin***, G. Borello (InterAC), 1st International Conference: Control and diagnostics in automotive applications – October 3-4, 1996 – Genova, Italy
  - ***Prediction and control of structure borne noise transfers in vehicles using SEA***, G. Borello (InterAC), Euro-Noise – October 4-7, 1998 – Munich, Germany
  - ***Test-based simulation bridges the gaps between test and CAE for spacecraft vibro-acoustic***, C. Kaminsky (VASci), G. Borello (InterAC) 19th Aerospace Testing Seminar – October 2-5, 2000 – Manhattan Beach, CA, USA
- Nevertheless ESEA needs hardware or prototype to work on
- VSEA was first simply ESEA applied to FE-generated FRF and SEA network can be created without any hardware
  - ***Virtual SEA: mid-frequency structure-borne noise modeling based on Finite Element Analysis,*** G. Borello (InterAC), L. Gagliardini, L. Houillon (PSA), L. Petrinelli (Geci Systems), SAE Noise and Vibration Conference – May 6-8, 2003 – Traverse City, Michigan, USA
- VSEA has evolved to a more complex theory in the last past 10 years to take into account specificity of FEM model
  - ***Virtual SEA: towards an industrial process***, G. Borello (InterAC), L. Gagliardini, D. Thenail (PSA), SAE Noise and Vibration Conference – May 15-17, 2007 – Saint Charles, Ill, USA

- VSEA subsystems can replaced analytical representations or complement them in frequency regions of strong dynamical morphing



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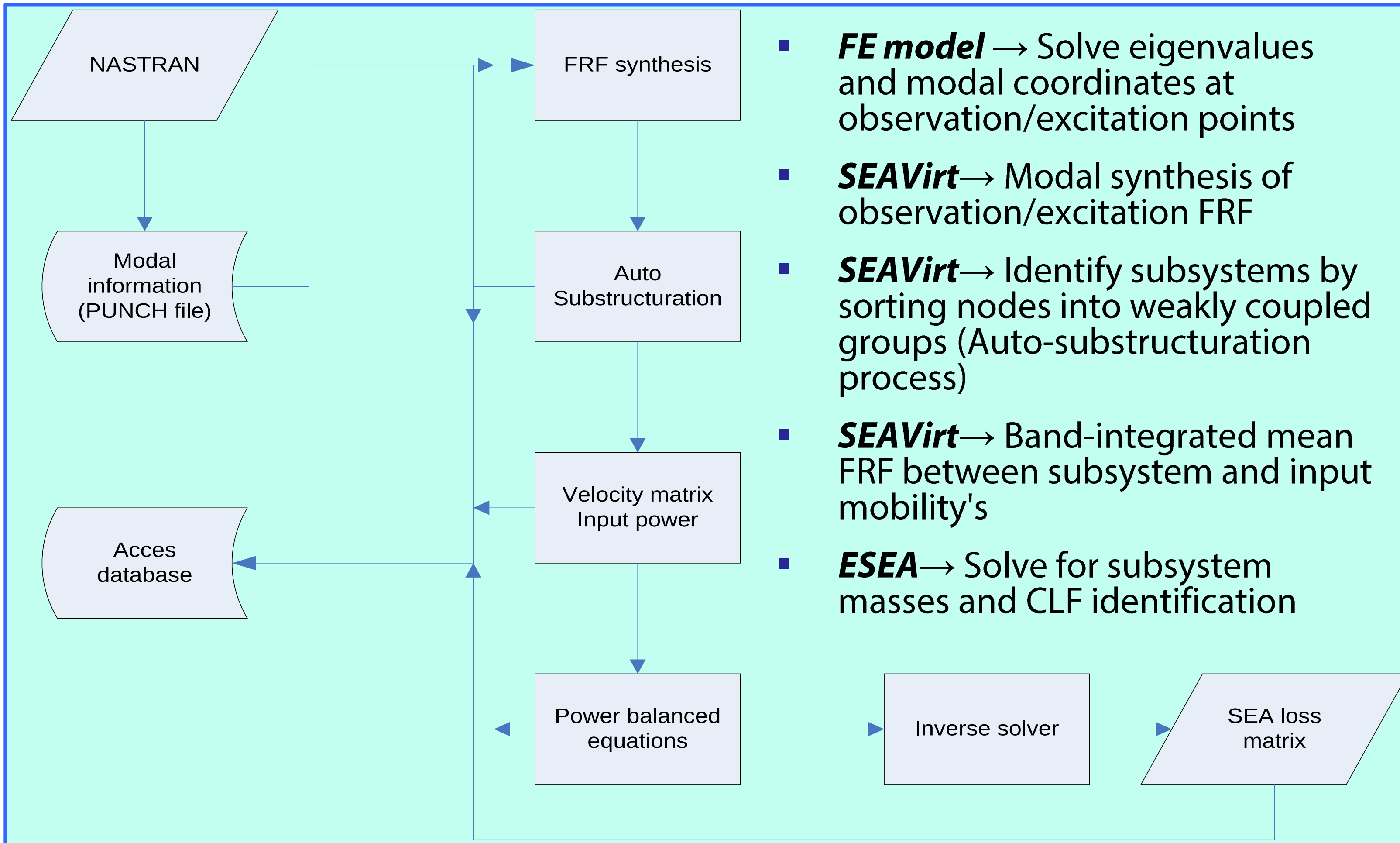
VSEA Work Flow

FEM model is excited by point forces and FRF is computed at a discrete number of observation nodes then SEA parameters are identified using inverse solver similar to ESEA



- ***FE model*** → Solve eigenvalues and modal coordinates at observation/excitation points
- ***SEAVirt***→ Modal synthesis of observation/excitation FRF
- ***SEAVirt***→ Identify subsystems by sorting nodes into weakly coupled groups (Auto-substructuration process)
- ***SEAVirt***→ Band-integrated mean FRF between subsystem and input mobility's
- ***ESEA***→ Solve for subsystem masses and CLF identification

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The VSEA Method

- *Eij* transfer energy is estimated from a structural FEM model using modal synthesis and the loss-matrix is estimated by inverse SEA method
- The automation of the FE-post-processing inverse SEA sequence has been called **Virtual SEA** or **VSEA** which transforms FE FRF into SEA network using following steps

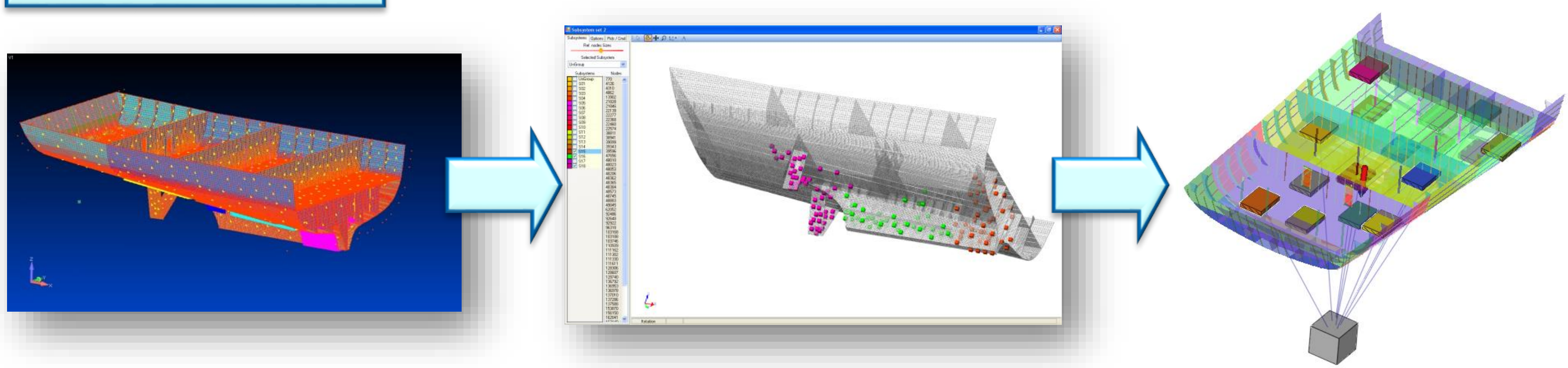
Band integrated FRF matrix generation when exciting FEM model with random point-forces on a grid of reference nodes

Auto-substructuration

Grouping nodes to create subsystems

Compression into Mean FRF/subsystem matrix

Inverse technique to identify SEA parameters



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The VSEA Method

- **Generating VSEA transfer functions**
  - Extraction of FEM modal amplitude at grid nodes and fast modal synthesis of nodal transfer velocity tensor assuming rain-on-roof excitation and global frequency dependent DLF

$$\left\{-\mathbf{M}\omega^2 + \mathbf{K}\right\} \mathrm{X} = \lambda\, \mathrm{X} \Rightarrow \bar{\bar{\mathbf{V}}}_{\mathbf{kl}} = \sum_i \frac{i\omega \mathrm{X}_i\left(x_k\right)\mathrm{X}_i^T\left(x_l\right)}{\lambda_i - \omega^2 + i\eta\omega\sqrt{\lambda_i}}$$

  - Band integration of $\bar{\bar{\mathbf{V}}}_{\mathbf{kl}}^2$ and $\mathrm{Re}\left\{\bar{\bar{\mathbf{V}}}_{\mathbf{ll}}\right\}$ then projection in largest eigenvalue direction of local input mobility tensor $\mathrm{Re}\left\{\bar{\bar{\mathbf{V}}}_{\mathbf{ll}}\right\}$

- **Auto-partitioning of the transfer velocity matrix (identify subsystems)**
  - Iterative peripheral attraction algorithm to sort nodes into weakly coupled groups
  - Discrimination of groups by calculation of the group entropy

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The VSEA Method

- **Identifying SEA parameters (VSEA inverse SEA solver)**
  - Identification of modal density as damping is a known quantity
  - Identification of CLF at each SEA junction
  - Monte-Carlo perturbations for variance estimate
  - Elimination of low-powered connections
  - Model reconstruction performance (measure the loss of information in the compression process)
- **Providing a wavenumber to VSEA subsystems**
  - A surface area is allocated to VSEA subsystems
  - The mean statistical wavenumber is derived from modal density
- **SEA hybridization of VSEA and ASEA subsystems**
  - Any analytical subsystems may be coupled with any VSEA subsystems as CLF are dependent of wavenumbers in both ASEA and VSEA subsystems
  - In particular, adding analytical fluid cavities is achieved computing radiation efficiency of VSEA structural subsystems
  - Radiation of structures is estimated from their radiated power flow

$$\mathbf{n} = \mathbf{E}^{(r)-1} \cdot \frac{\mathbf{1}}{\omega_c}$$

$$\boldsymbol{\eta}_{i\alpha} = \mathbf{E}_{\alpha i}^{(r)-1} \cdot \frac{\mathbf{1}}{\omega_c \mathbf{e}_{ii}}$$

$$\mathbf{k}_{ss} = \sqrt{\frac{2\omega\mathbf{n}}{\mathbf{S}}}$$

$$\boldsymbol{\eta}_{\mathbf{rad}} = \frac{\boldsymbol{\sigma\rho c}\mathbf{S}}{\boldsymbol{\omega}\mathbf{m}}$$

$$\boldsymbol{\Pi}_{\mathbf{rad}} = \boldsymbol{\eta}_{\mathbf{rad}}\boldsymbol{\omega}\mathbf{E}$$

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Key Technology in SEAVirt

- **Fast FRF generator** based on modal synthesis
- **Auto-creation of subsystems** from FRF matrix
- Identification protocol suitable for non-homogeneous subsystems
- Wave number identification for acoustic coupling and structural coupling
- Any VSEA subsystem can be coupled to any ASEA subsystem

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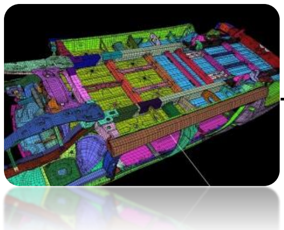
FRF Generator: Fast FRF Modal Synthesis



Nastran Modal solveur

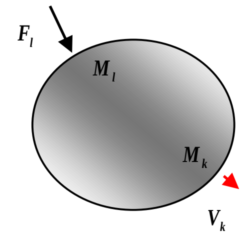
Punch file

SEAVirt FRF Synthesis

- **Band-avereged**
- **Band optimized**
- **Normal/normal**
- **Max Exc/Max Obs.**

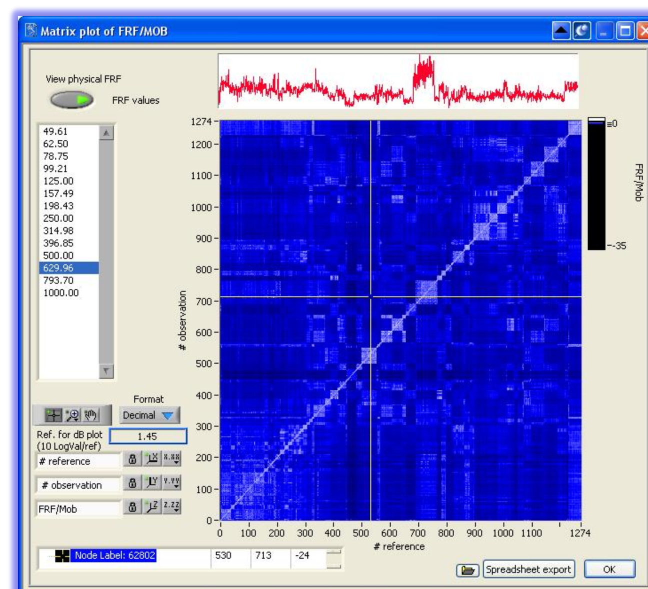
$$U_k^l = H_{kl} F_l = \sum_i H_i \psi_{ikl}$$

V²(o,e) Real(Me)



$$\begin{bmatrix} U_{x_k} \\ U_{y_k} \\ U_{z_k} \end{bmatrix}_{kl} = \begin{bmatrix} H_{x_k x_l} & H_{x_k y_l} & H_{x_k z_l} \\ H_{y_k x_l} & H_{y_k y_l} & H_{y_k z_l} \\ H_{z_k x_l} & H_{z_k y_l} & H_{z_k z_l} \end{bmatrix} \begin{bmatrix} F_{x_l} \\ F_{y_l} \\ F_{z_l} \end{bmatrix}$$

Tensor toe between two nodes



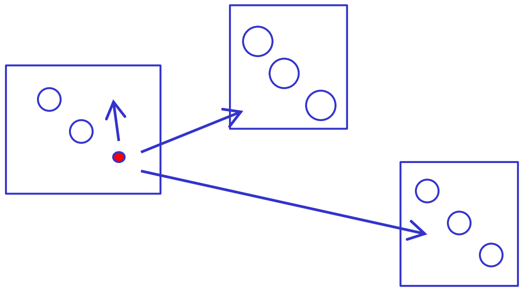
FRF solver is designed for speed with several settable algorithms

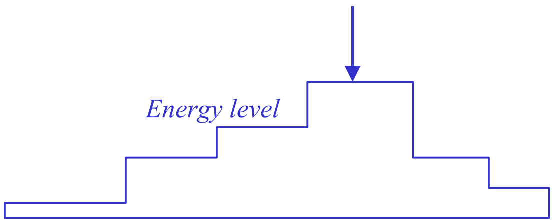
1. *Compute all FRF tensors Toe between nodes of reference grid*
2. *Project Toe in the direction of max input power at o and at e nodes*
3. *Band-averaging of the projected FRF node-to-node matrix*

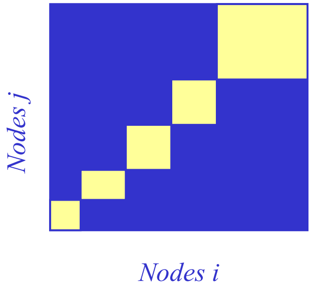
Projected FRF matrix

Auto-substructuration (Identify Subsystems)

- The original algorithm for self-partitioning the system into SEA subsystems is based on intuitive definition of "**weak coupling**" assumption
  - A subsystem is **weakly** coupled with its neighbors if it takes more energy when excited internally than externally
  - A node is **attracted** by a subsystem if its local energy is the largest when excitation is applied in the related subsystem
- Nodes are then submitted to attraction by subsystems
- From initial a priori partition (based on user-defined FRF matrix threshold), the iterative attractive peripheral algorithm moves nodes to most attractive subsystems. In the next iteration, it computes again the energy state of the system and the attraction "forces" and moves again nodes and so on…
- Due to dependence of initial partition on final result, range of thresholds are swept given a related subsystem decomposition leading to series of SEA partitions
- Among the series, some partitions are better than others: partition must be SEA invertible and the diagonal of the SEA transfer energy matrix should be the highest with low off-diagonal terms, condition mathematically given by the condition number of the matrix



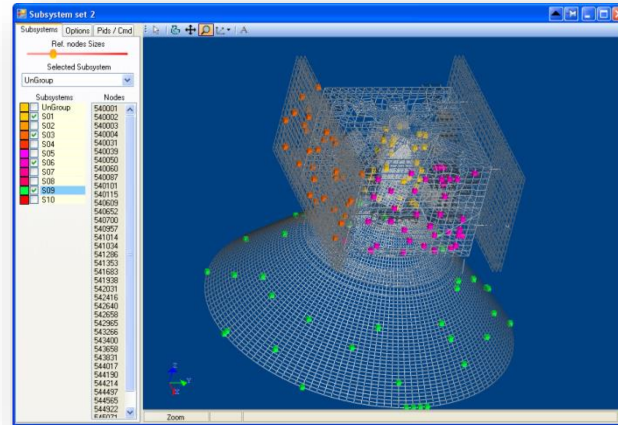


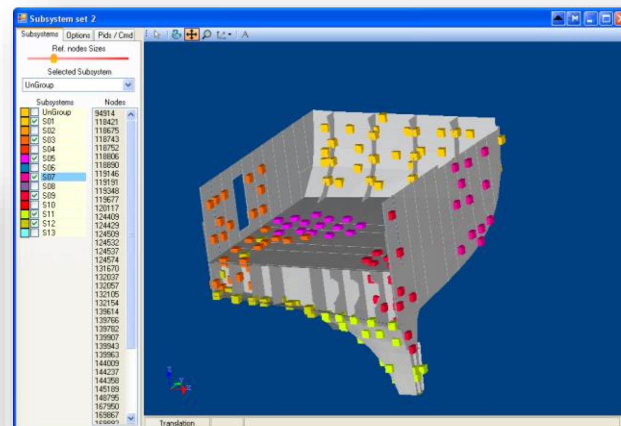


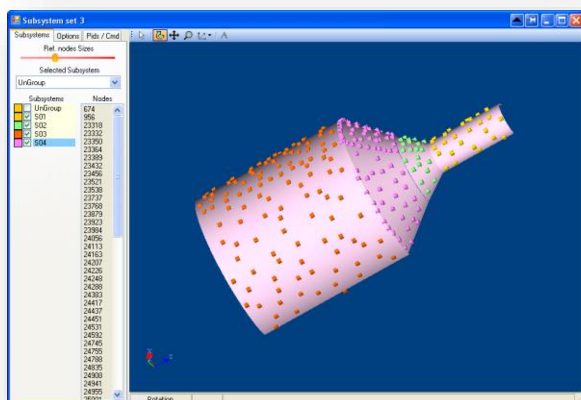
Threshold FRF matrix

Auto-substructuration (Identify Subsystems)

- Specific sort algorithms to group nodes into **weakly coupled** regions
- **Principle**: the SEA state of energy maximizes the energy gap between subsystems
- The FRF matrix is iteratively re-organized to maximize the energy gap between groups
- At the end the best fitted groups are found and identified as SEA subsystems







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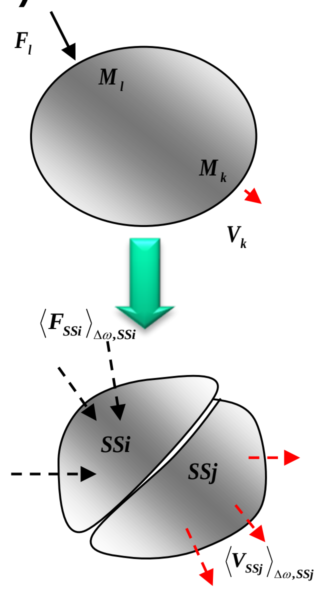
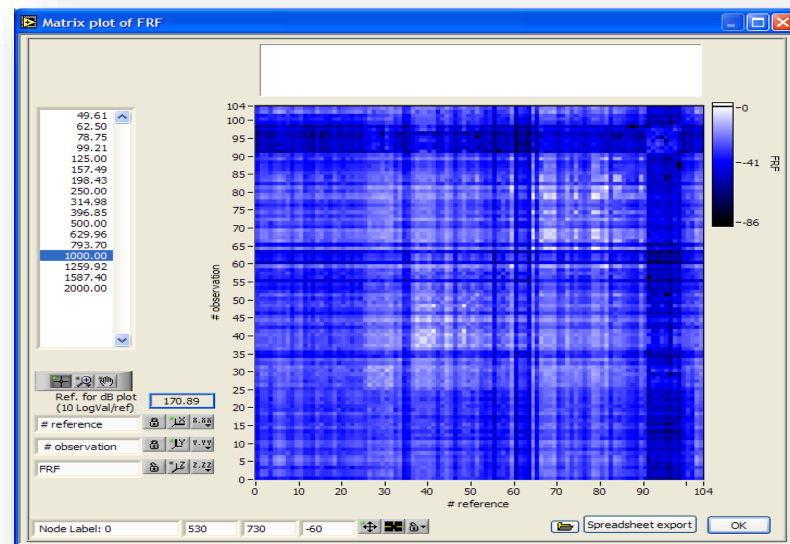
Band-Averaging and Compression on Subsystems

- **Synthesized FE-point-to-point FRF (spatial & frequency phase)**

FE-FRF

SEA-FRF (partition)





Compressed SEA FRF matrix

- **SEA FRF : mean FRF between subsystem integrated over 1/nth octave bands**

$$\left\langle V^2{}_{ji}(\omega_c,\Delta\omega)\right\rangle = \frac{1}{\Delta\omega}\int_{\Delta\omega}\left\{\frac{1}{N_k N_l}\sum_{k,k\in SSi}\ \sum_{l,j\in SSj} V^2{}_{kl}(\omega)\,d\omega\right\}$$

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The Reduced SEA Transfer Matrix

- In standard ESEA, the power balanced equations are expressed in term of mass and velocity²
- In non-homogeneous systems, large standard deviation of input mobility and mean velocity² can be obtained depending on observation points
- To minimize this point dependence in the resulting SEA model, velocity² matrix is normalized by mobility of input and output points before compression
- In a non-homogeneous subsystem *K*, modal density, mass and input mobility are related at each point *i* by:

$$N_i = 4 m_i Y_i$$

- The power flow equations can thus be expressed in term of modal density and reduced velocity matrix which is proportional to the local modal energy ε_{ij}:

$$Y_{iL} = \sum_{K/j \in K} \eta_K \omega \left\langle m_j V_{ij}^2 \right\rangle_{j \in K} \Rightarrow 1 = \sum_{K/j \in K} \eta_K \omega \cdot \left\langle N_j \right\rangle \left\langle \frac{V_{ji}^2}{4 Y_j Y_i} \right\rangle_{j \in K}$$

$$\varepsilon_{ij} = \frac{V_{ji}^2}{4 Y_j Y_i}$$

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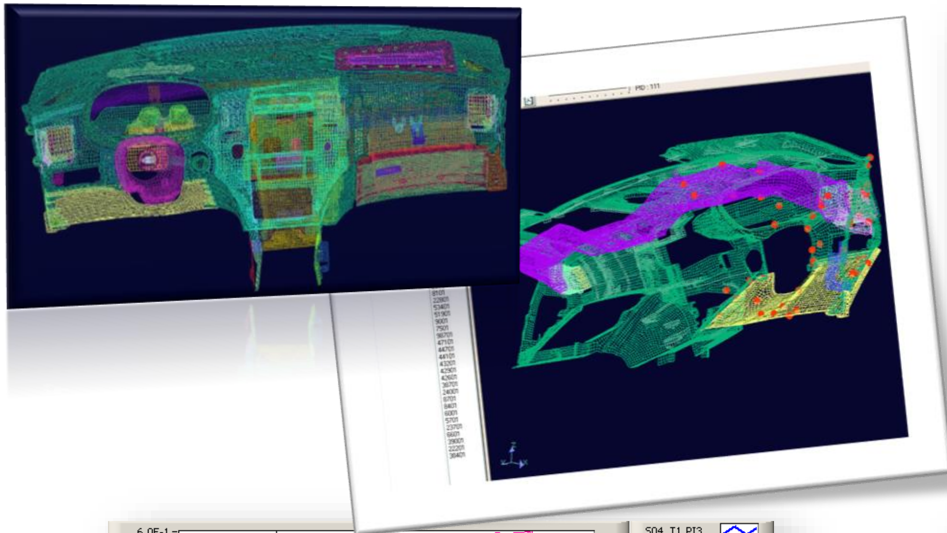
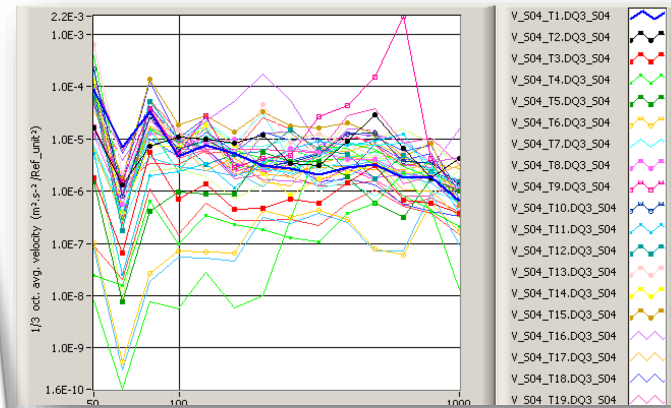
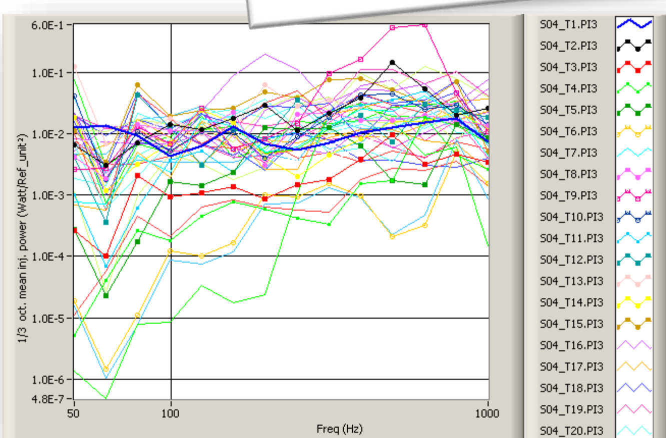
Reducing Point Dependence on Observation Nodes

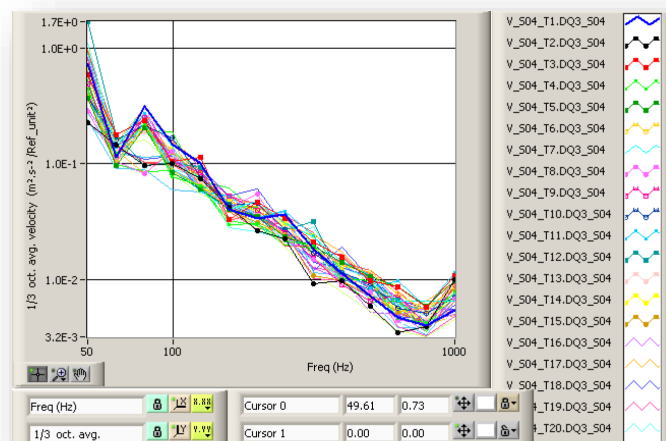




Velocity²



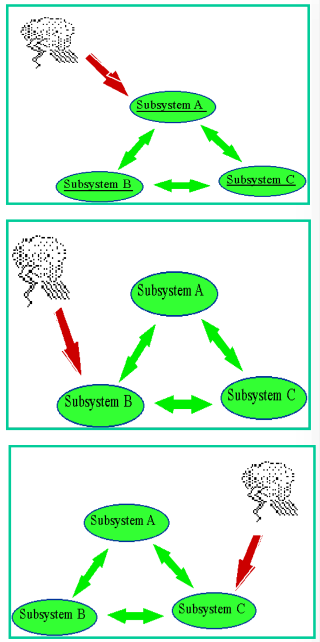
Mobility (Re) Yi

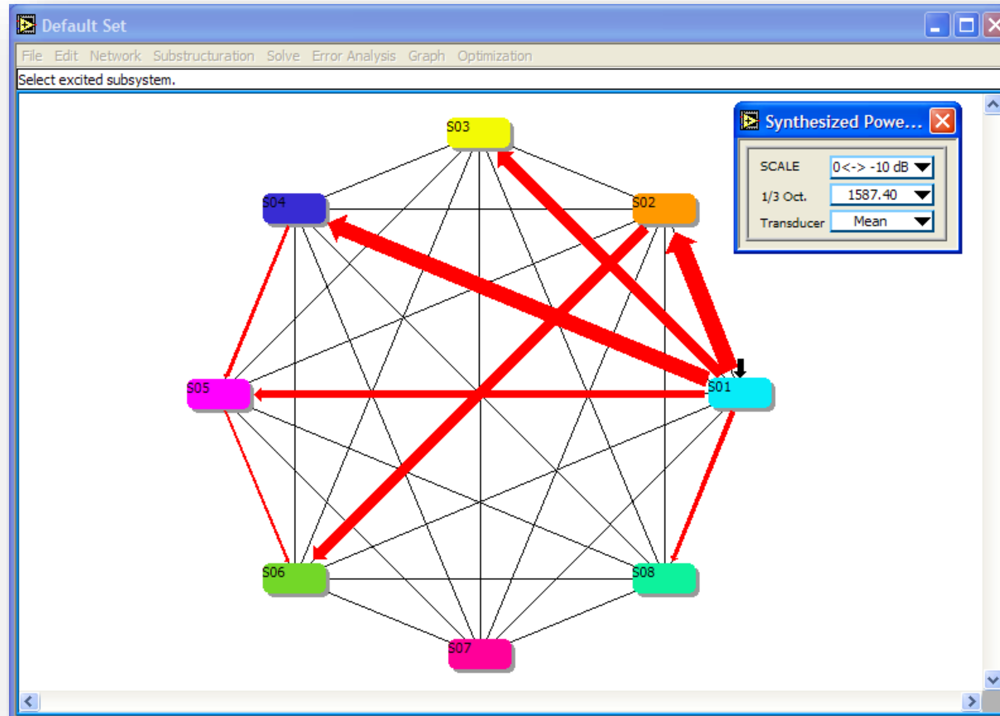


Velocity²/4YiYj

SEA Inverse Solver

- Identify all SEA loss matrix parameters from power-balanced equations





$$P_{ji} = \omega_c \left\{ \eta_{ji} E_j - \eta_{ij} E_i \right\}$$

$$\Pi_j = \omega_c L_{ji} \ E_i$$

$$L^{-1} = \begin{bmatrix} \eta_1 + \sum_j \eta_{1j} & \cdots & -\eta_{j1} & \cdots \\ \cdots & \eta_i + \sum_j \eta_{ij} & -\eta_{ji} & \cdots \\ -\eta_{1j} & -\eta_{ij} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}^{-1}$$

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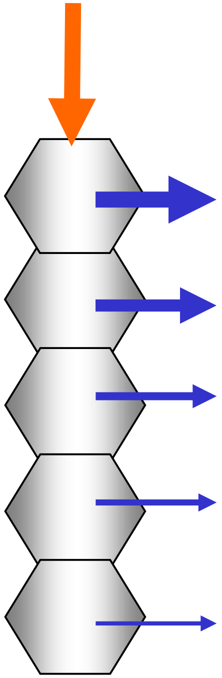
Solve for Modal Density

- **Standard Solve** with square modal energy/subsystem matrix

$$\frac{1}{\omega_c}\begin{bmatrix}1\\ \dots\\ 1\end{bmatrix}=\begin{bmatrix}\eta_1\varepsilon_{11}^2 & \dots & \eta_N\varepsilon_{N1}^2\\ \dots & \dots & \dots\\ \eta_1\varepsilon_{1N}^2 & \dots & \eta_N\varepsilon_{NN}^2\end{bmatrix}\cdot\begin{bmatrix}N_1\\ \dots\\ N_N\end{bmatrix}$$

$$P_{inj}=\sum_n P_{diss}$$



- **SVD solve** with rectangular modal energy/node matrix

$$\frac{1}{\omega_c}\begin{bmatrix}1\\ \dots\\ 1\\ \dots\\ 1\end{bmatrix}=\begin{bmatrix}\eta_1\varepsilon_{11}^2 & \dots & \eta_N\varepsilon_{N1}^2\\ \dots & \dots & \dots\\ \eta_1\varepsilon_{1l}^2 & \dots & \eta_N\varepsilon_{Nl}^2\\ \dots & \dots & \dots\\ \eta_1\varepsilon_{1N}^2 & \dots & \eta_N\varepsilon_{NN}^2\end{bmatrix}\cdot\begin{bmatrix}N_1\\ \dots\\ N_l\\ \dots\\ N_N\end{bmatrix}$$

Solve for CLF

$$c_{ji} = N_j \varepsilon_{ji}$$

***Standard solve**: square matrix*

$$\left[\eta_{ki}\right]_{k\in\{\alpha,N_i\},k\neq i} = \begin{bmatrix} \frac{c_{\alpha i}}{\varepsilon_{ii}} - \frac{c_{\alpha\alpha}}{\varepsilon_{i\alpha}} & \dots & \frac{c_{N_i i}}{c_{ii}} - \frac{c_{N_i\alpha}}{c_{i\alpha}} \\ \dots & \frac{c_{\beta i}}{c_{ii}} - \frac{c_{\beta k}}{c_{ik}} & \dots \\ \frac{c_{\alpha i}}{c_{ii}} - \frac{c_{\alpha N_i}}{c_{iN_i}} & \dots & \frac{c_{N_i i}}{c_{ii}} - \frac{c_{N_i N_i}}{c_{iN_i}} \end{bmatrix}^{-1} \cdot \begin{bmatrix} -\frac{1}{c_{ii}} \\ -\frac{1}{c_{ii}} \\ -\frac{1}{c_{ii}} \end{bmatrix}$$

***SVD solve**: rectangular matrix*

$$\left[\eta_{ki}\right]_{k\in\{\alpha,N_i\},k\neq i} = \begin{bmatrix} \left[\frac{c_{\alpha i}}{c_{ii}} - \frac{c_{\alpha\alpha}}{c_{i\alpha}}\right]_1 & \dots & \left[\frac{c_{N_i i}}{c_{ii}} - \frac{c_{N_i\alpha}}{c_{i\alpha}}\right]_1 \\ \dots & \left[\frac{c_{\beta i}}{c_{ii}} - \frac{c_{\beta k}}{c_{ik}}\right]_l & \dots \\ \left[\frac{c_{\alpha i}}{c_{ii}} - \frac{c_{\alpha N_i}}{c_{iN_i}}\right]_L & \dots & \left[\frac{c_{N_i i}}{c_{ii}} - \frac{c_{N_i N_i}}{c_{iN_i}}\right]_L \end{bmatrix}^{-1*} \cdot \begin{bmatrix} -\frac{1}{c_{ii1}} \\ -\frac{1}{c_{iil}} \\ -\frac{1}{c_{iiL}} \end{bmatrix}$$

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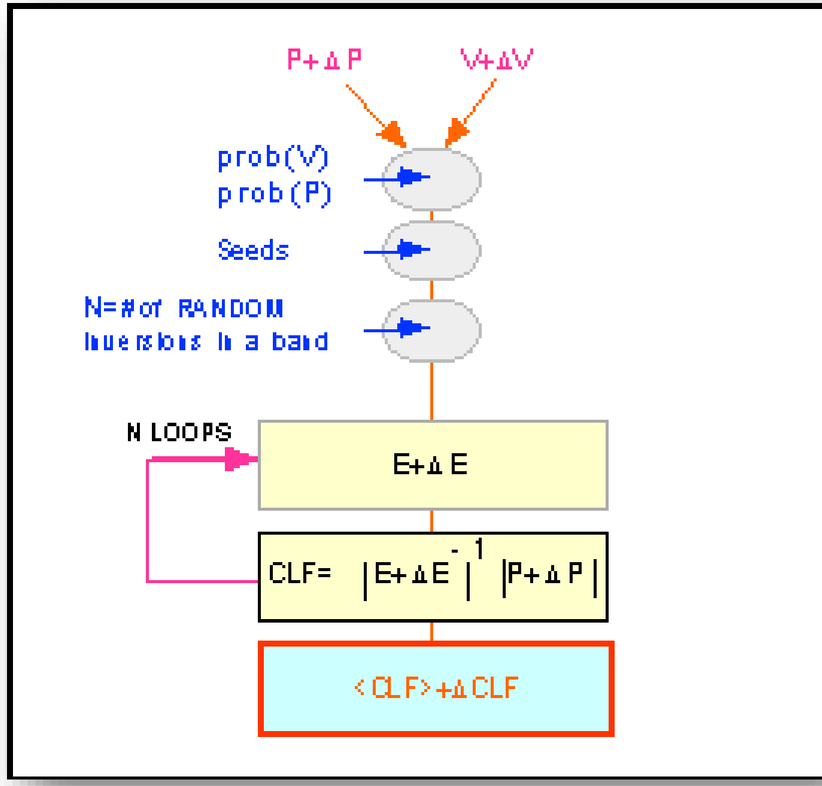
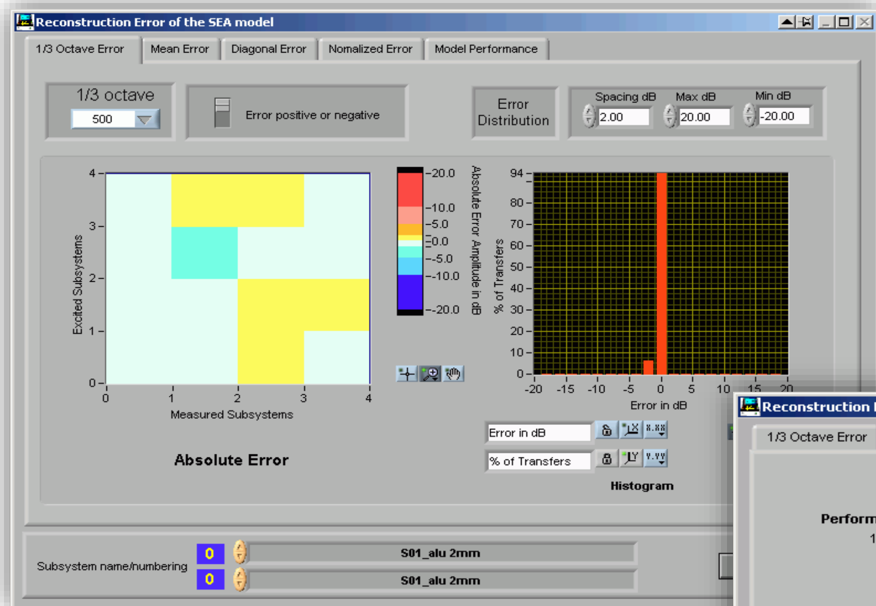
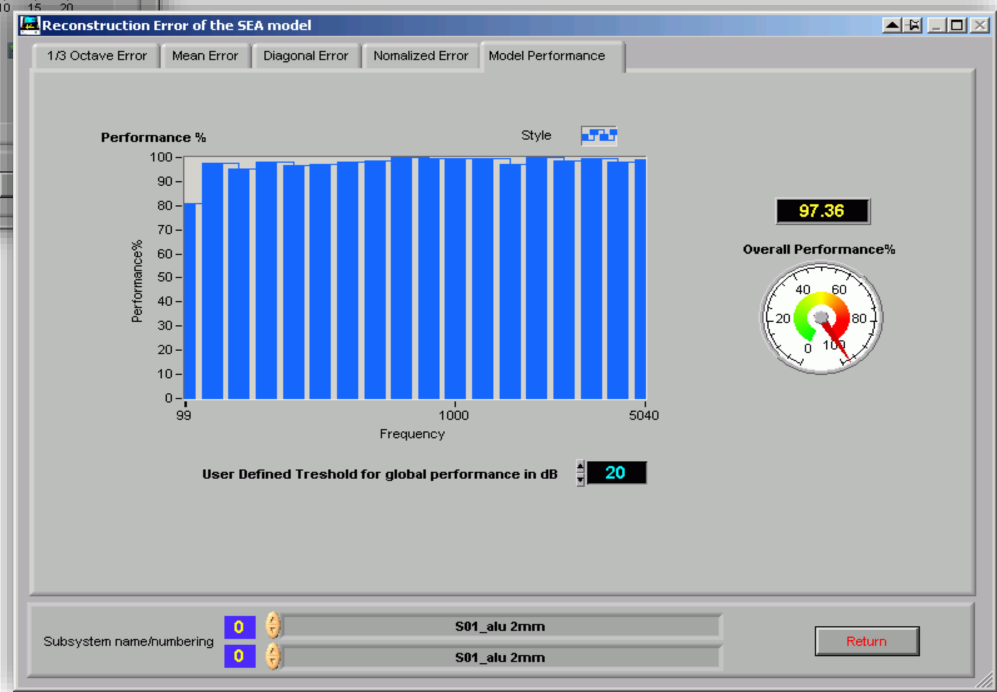
Exploring the Parameter Space with Monte Carlo Solve

- Random perturbation of input database (Gaussian or uniform)
- Rejection of non-physical solutions



Analyzing SEA Model Quality





$$\tilde{V}^2 = L^{-1}P$$

$$Error = \frac{V^2 - \tilde{V}^2}{V^2}$$

Reconstruction error on quadratic transfer velocity and SEA Model Performance Index

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Point-Transfer Reconstruction

- The point-point transfers are reconstructed from the statistical modal transfer energy as (*m* and *n* indexes of subsystems, *i* and *j* points)

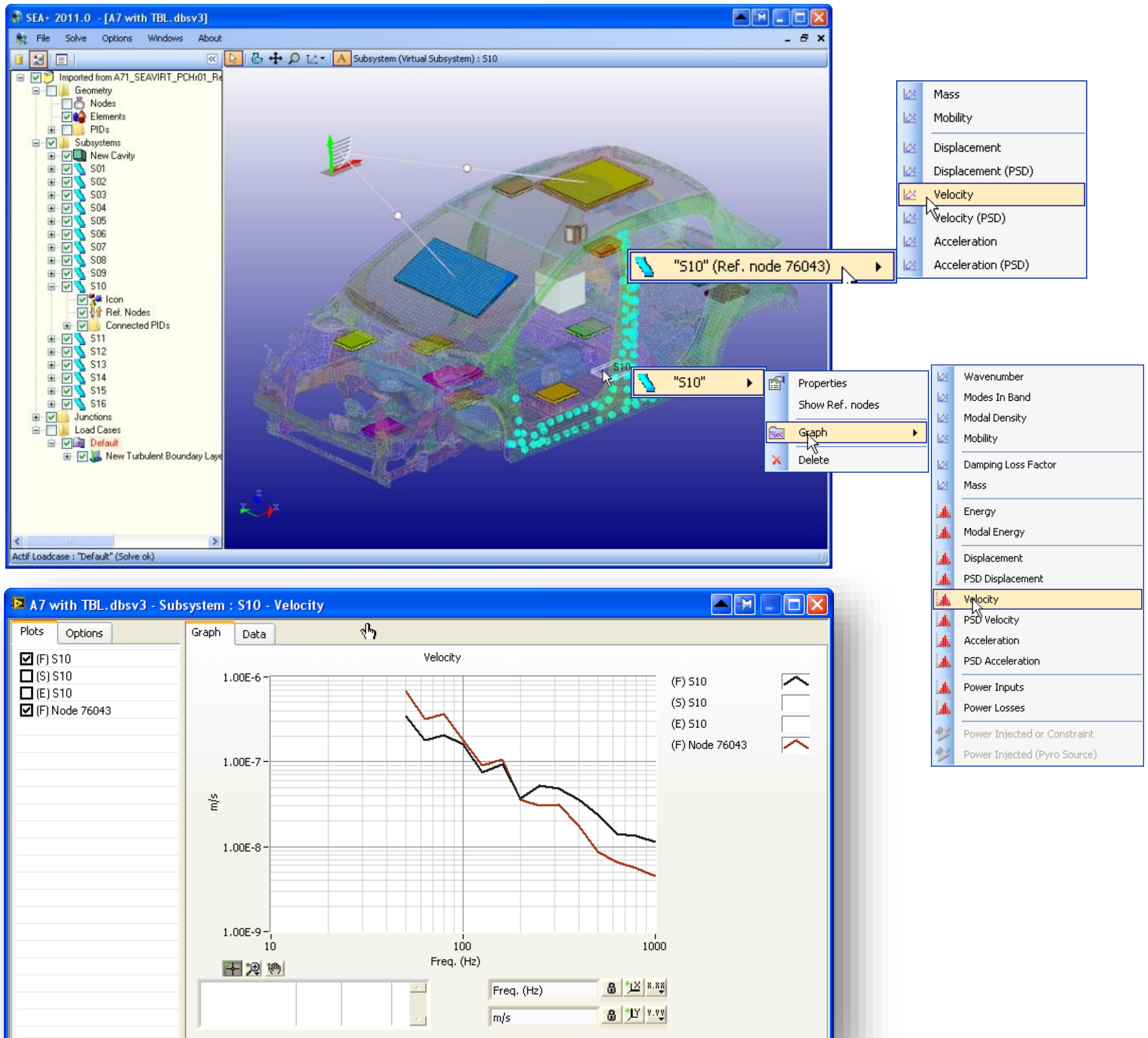
$$V_{mn}^2\left(x_i, x_j\right) = 4Y_iY_j\left\langle \varepsilon_{mn} \right\rangle F_i^2 = 4Y_iY_j\frac{L^{-1}\Pi_i}{\omega}F_i^2$$

- SEAVirt data flow appears as a **compression algorithm** of FE dynamical information able to restitute FE FRF within a given confidence level from a small square *L* matrix and the input mobility vector

- As soon as the *L* matrix (loss factor matrix) is identified, the response at any point of structure can be predicted by only knowing input mobility information at excitation and observation points

3D-Response Field Reconstruction at VSEA Nodes

- VSEA models provide SEA outputs at all reference nodes for all kind of excitations



Car Chassis Results

- ASEA model only converges above 3 to 4 kHz
- Morphing frequency domain 200-2000 Hz
- Perfect predictability of VSEA model up to 2 kHz (blind construction)
- Validation with measurement performed by Laser velocimetry for accurate mean velocity estimate (PSA measurement)
- Adaptive substructuration to frequency band for best performance
  - LF model 200-800 Hz
  - HF model 500-2000 Hz

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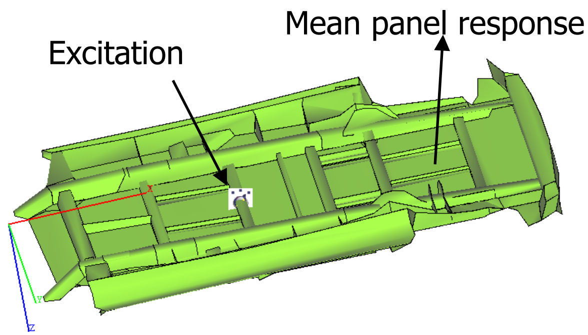
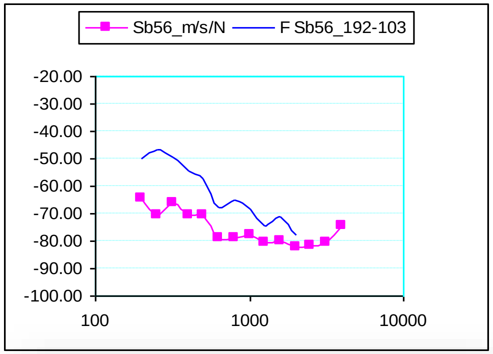
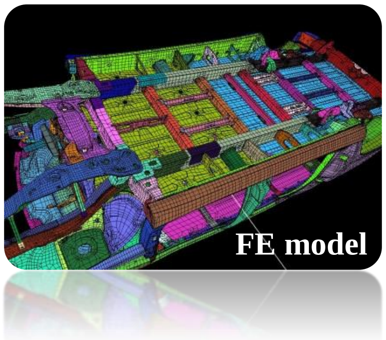
Application Example: Car Chassis Structure Borne Sound

Analytical SEA model

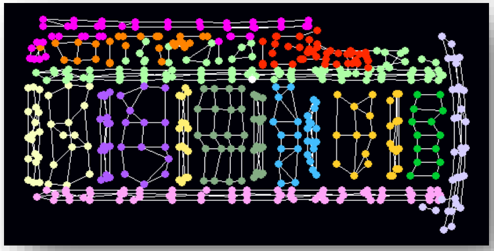




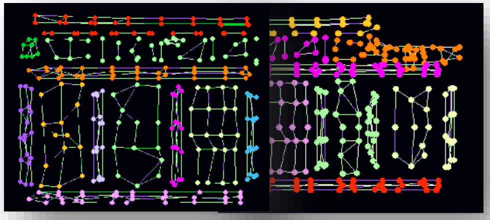
Virtual SEA model

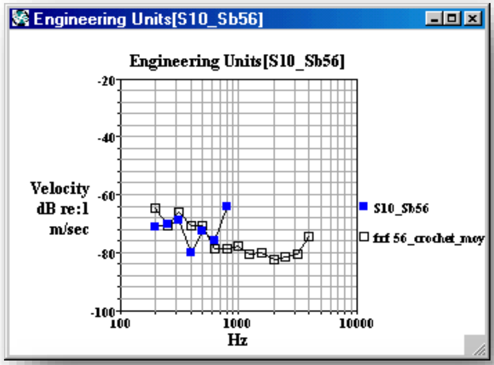


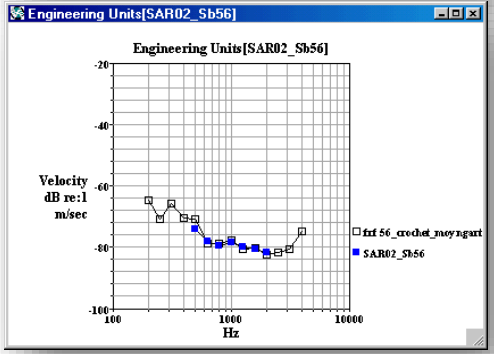
LF model 200-800 Hz



HF model 500-2000 Hz







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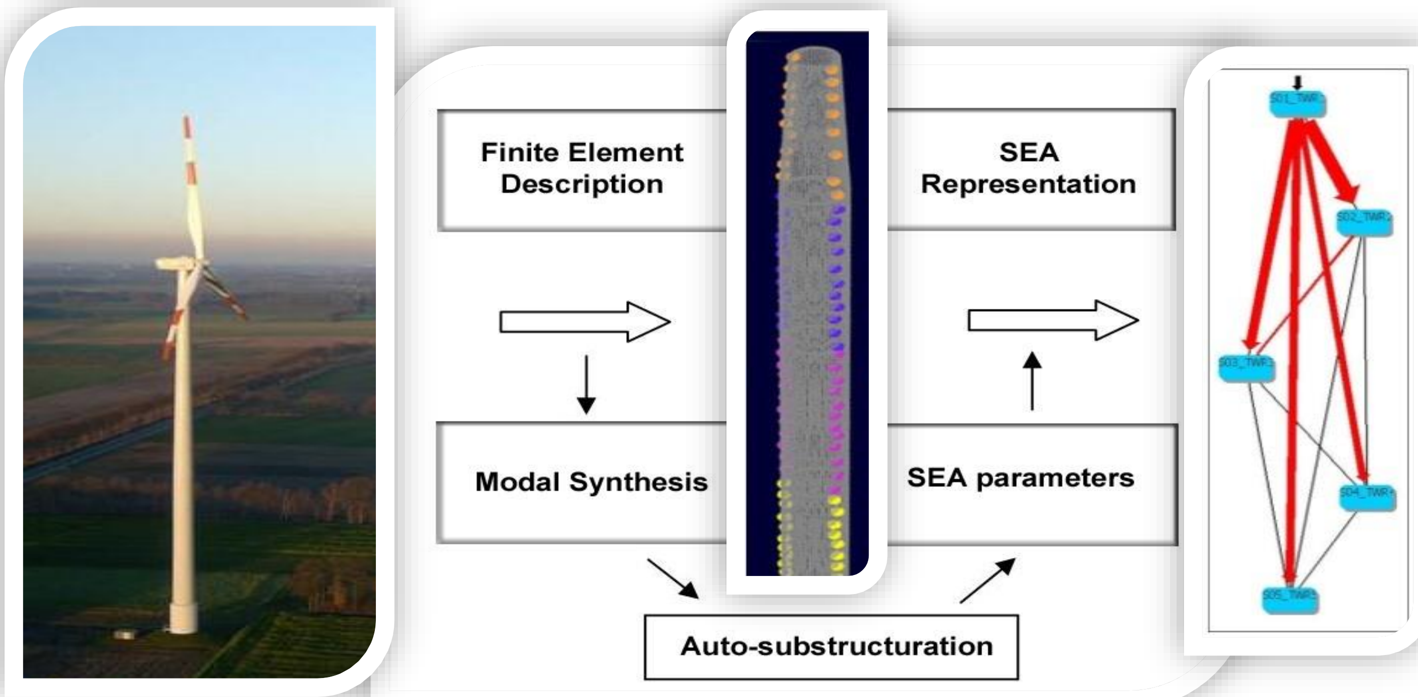
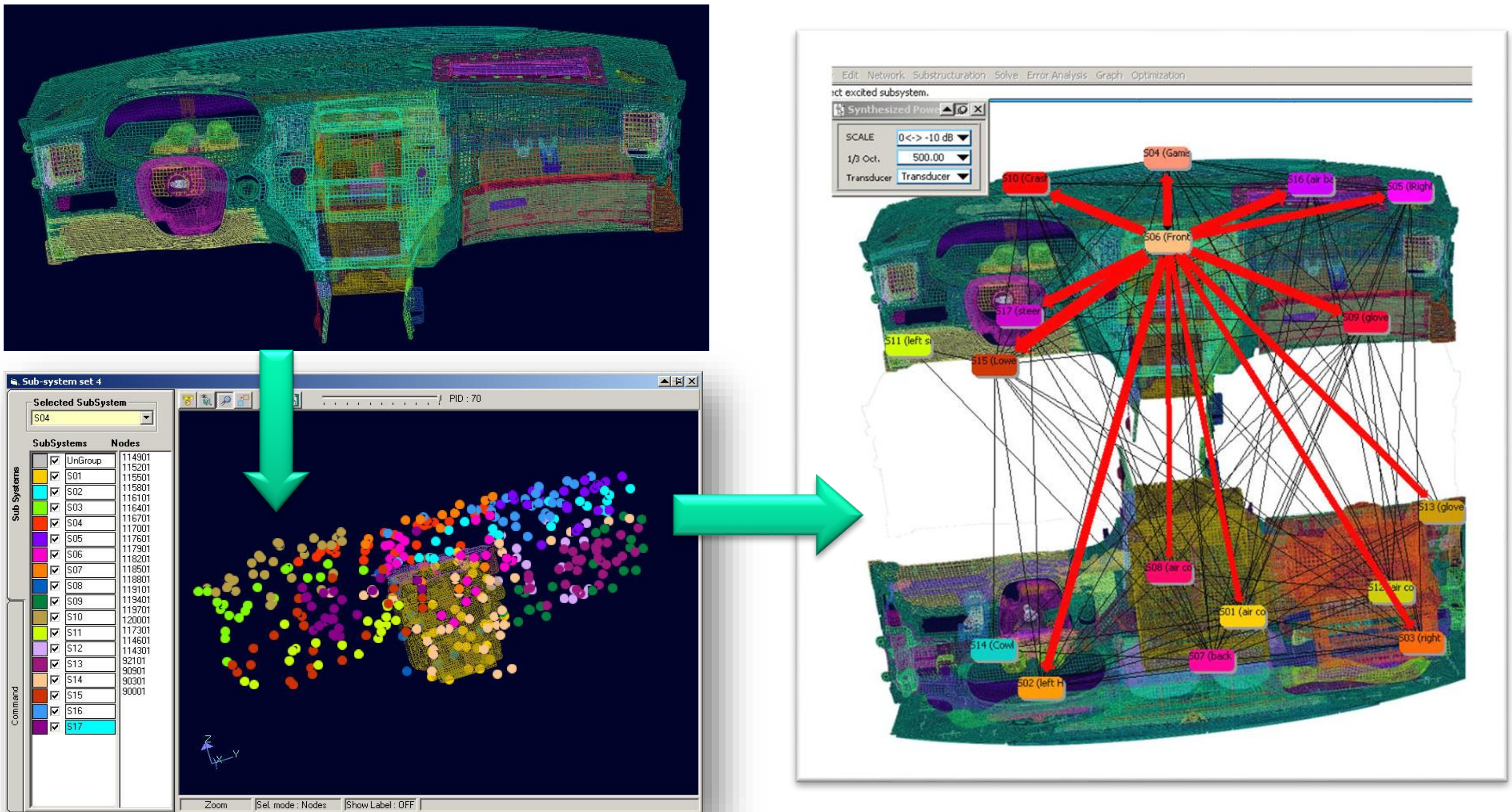
Application Example: Wind Turbine Modeling

- **Goal**: prediction of tower radiation by analytical SEA → need to know the vibration drop of energy between tower segments
- **Outputs**: VSEA provides the best subsystem decomposition and the CLF between subsystems



Application Example: Cockpit Structural Transmissibility

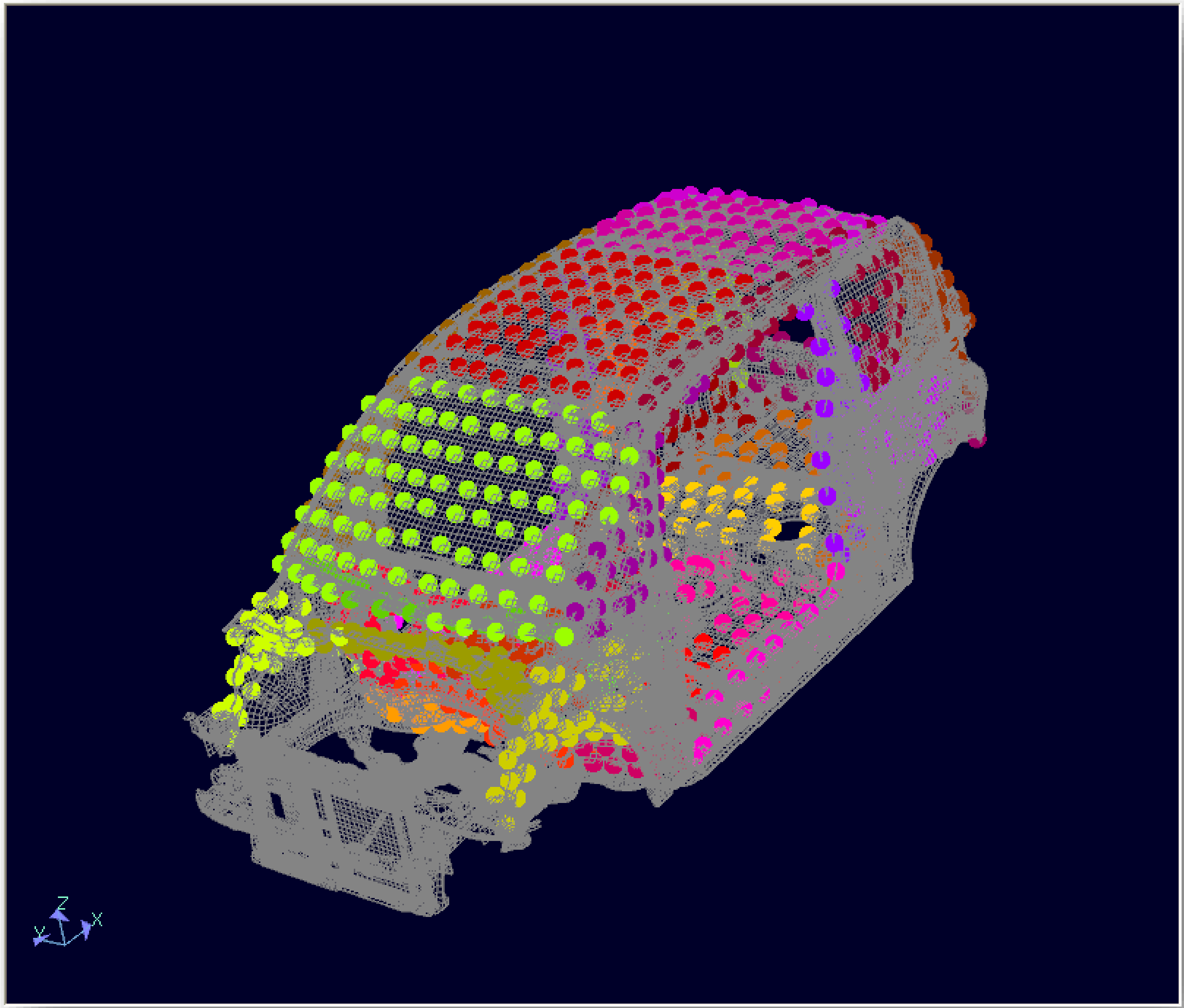
- From customer's FEM model, analysis of cockpit structural subsystem decomposition to guide ESEA



Application Example: Body-in-White Analysis

PSA test case :

Computation up to 1000 Hz, 5000 modes, 1150 nodes



Structural VSEA Subsystems Linked to Acoustic Subsystems

- Enclosed FE meshed cavities mixed with structures (not available in standard)
  - Compatible with present technology
  - With {f, u} formulation of fluid structure interaction (potential, displacement), the acoustic excitation is a unit displacement applied to an acoustic node. The SEAVirt FRF solver must only known whether a node is in the fluid or the structure to scale the excitation and response terms correctly
- Enclosed analytical cavities
  - The CLF is estimated from the radiation efficiency that requires only to know wave number of the structural subsystem and related modal densities
- In SEAVirt, the modal density is converted into wave number following the relationship for shells:

$$k = \sqrt{\frac{2\omega \cdot N}{S}}$$

- Any VSEA subsystem can then be coupled with any ASEA subsystems

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VSEA: SEA Related Benefits

- The VSEA technology can create an SEA model from any FEM model
- VSEA provides accurate structure borne sound transmission models with no specific simplified assumption in the related SEA model
- Classical way of creating SEA models is improved:
  - Determination of subsystem partition and indirect coupling between subsystems
  - Asymptotic behavior of dynamic response is identified and can be used as template to scale analytical theory to extend prediction above FE limit
- All already created FE models can be re-used to generate VSEA models and overcome the traditional analytical modeling technique in SEA software as VSEA can be used at any scale
- SEAVirt data can be imported in SEA software to create hybrid SEA models
  - Above the morphing frequency domain, ASEA becomes predictive
  - Within the morphing domain, the VSEA model is replacing the ASEA model

VSEA: FEM Related Benefits

- SEAVirt can be also used as a data compressor of dynamical information in order to capture the main physics
- Retrieving point-to-point description of local vibration is possible with the mobility information
- It provides a procedure to quickly check FEM model in the upper limit of its frequency domain of validity
- Combining ESEA and VSEA affords a suitable method to validate dynamic modeling in the mid-frequency range
  - Junction modeling verification (point junction assumptions, impedance conditions…)
  - Construction validity (sandwich, double-wall…)

Validation on Numerical Test Cases

- VSEA is an easy way to verify SEA analytical modeling compared to related FEM
- VSEA may be consider as virtual testing and automatically provides modal density of local modes and their CLF in a FE model
- Applied to a simple configuration (curved-shell, coupling between simple panels) it provides with a high degree of confidence the related SEA parameters making possible to refine classical SEA modeling by taking into account actual shape effect
- In next slide VSEA virtual testing results are exploited for validating the analytical SEA kernel

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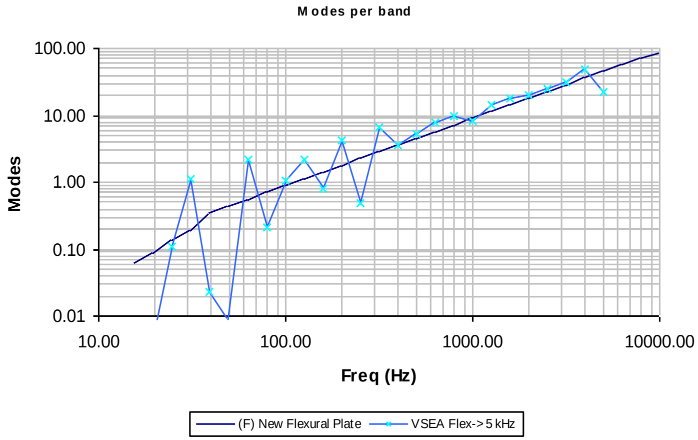
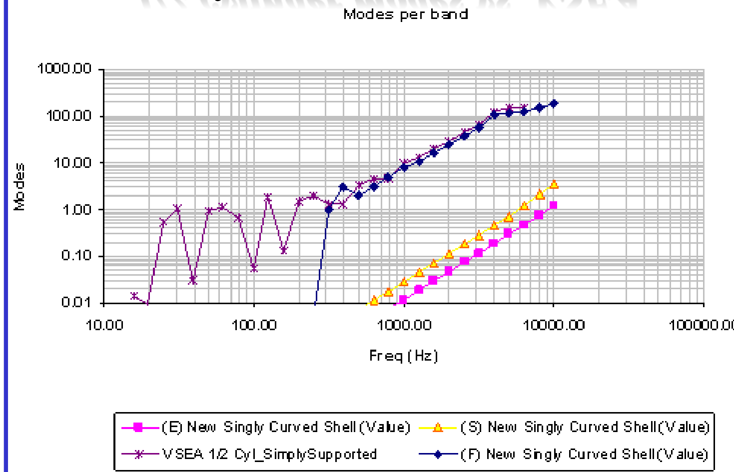
Numerical Test Cases: Modal Density Validation/VSEA

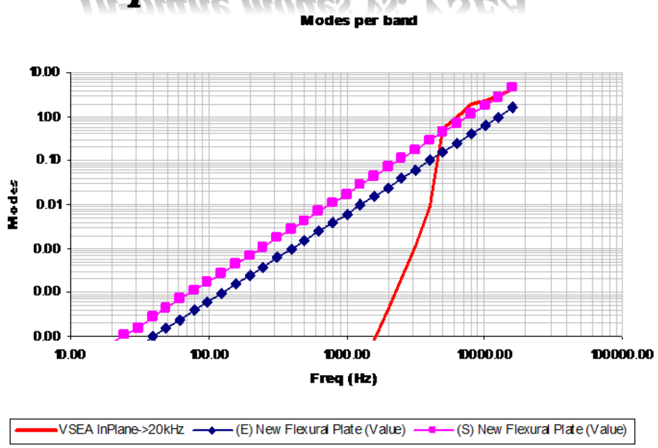
Plate flexural modes vs. VSEA



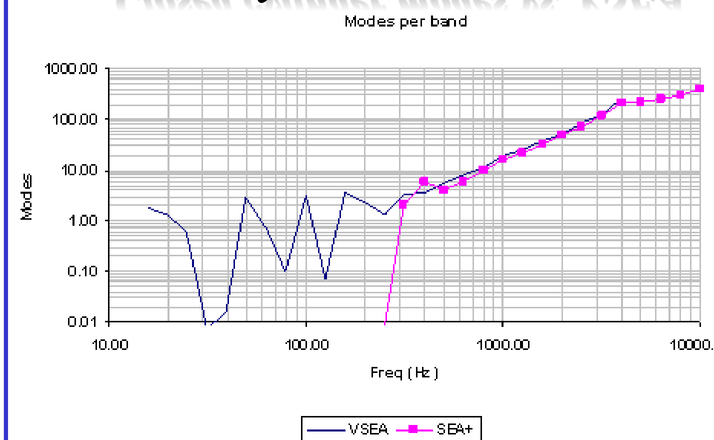
1/2 cylinder modes vs. VSEA



In-plane modes vs. VSEA



Closed cylinder modes vs. VSEA



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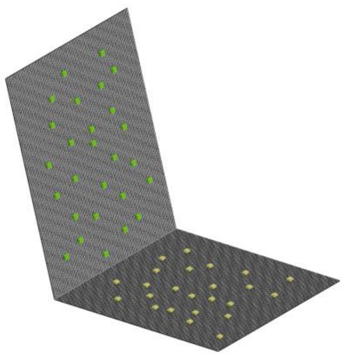
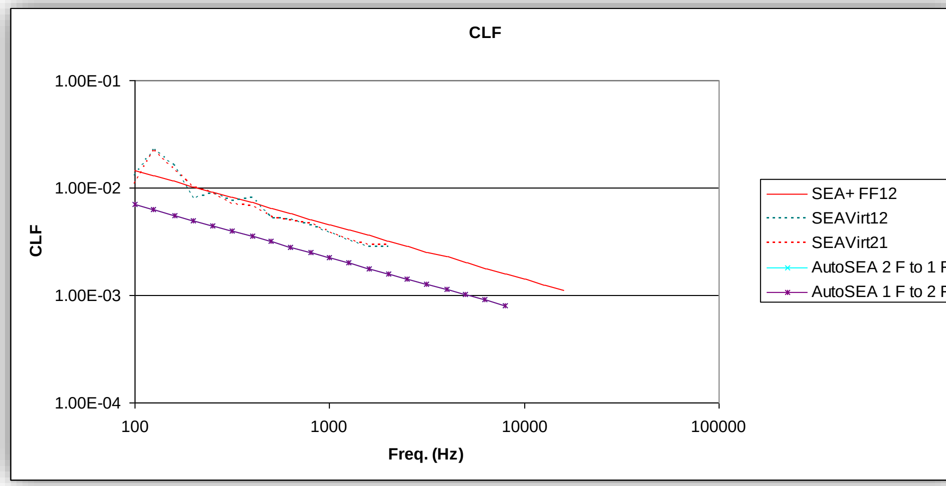
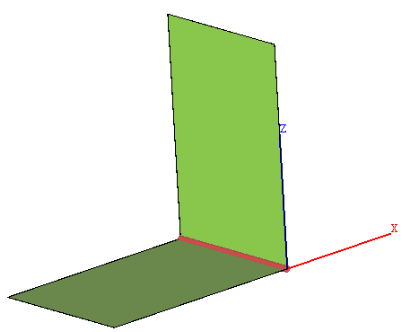
Numerical Test Cases: CLF Calculation/VSEA

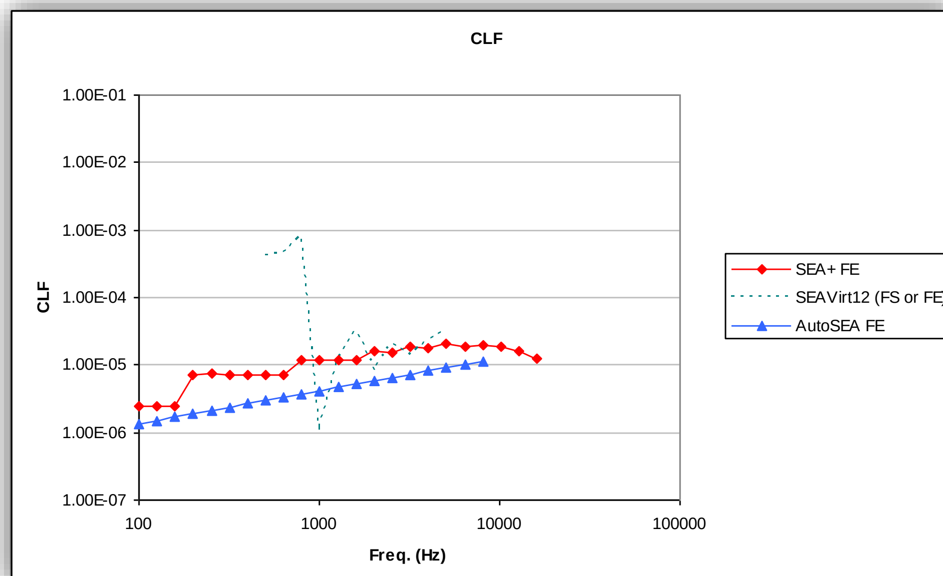
CLF flex-flex





CLF flex-extension





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